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Dispositional Logic

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1. Introduction

Mathematical analysis as we know it today is based almost entirely on two-valued logical systems in which every proposition is either true or false, with no gradation of truth allowed. However, this restriction is mitigated to some extent by allowing probability qualification, that is, assertions of the form: *The probability that proposition p is true is α, where α is a number in the interval* [0, 1].

The successes of mathematical analysis in science have led to an almost unquestioned assumption that the combination of two-valued logic and probability theory is all that is needed, especially in those branches of science, e.g., physics, chemistry, astronomy, analytical mechanics, control theory, etc., in which the systems under analysis are well-defined in the sense that their behavior may be described by a family of integro-differential or difference equations in which the stochastic components, if any, have known or measurable probability distributions. But what is questionable is the adequacy of two-valued-logic-based mathematics for the social sciences and, more generally, for the analysis of systems in which human judgment, perception, and emotions play an important role. The problem with such systems is that they are much too ill-defined to admit of characterization within any mathematical framework which is based on two-valued logic. This holds, in particular, for systems encountered in such fields as economics, psychology, sociology, medicine, etc. It is true that even in these fields, as in most fields of science, respect and recognition is accorded to those who create quantitative theories within the classical paradigm. But the extent to which such theories are relevant to real-world problems is a debatable thesis which cannot be supported in the affirmative if one expects a theory to have a predictive value.

Viewed against this background, fuzzy logic may be regarded as an alternative to two-valued logical systems—an alternative which aims at providing a model for modes of reasoning which are approximate rather than exact. Thus, in fuzzy logic, which is basically a generalization of multivalued logic, truth is not only a matter of degree—as it is in multivalued logic—but, more importantly, a fuzzy degree. The same applies to quantifiers, probabilities, possibilities and, more generally, to everything else. In this way, that is, by abandoning the concept of two-valuedness, fuzzy logic acquires the capability to model cognitive phenomena which are too complex or too ill-defined to be amenable to analysis by traditional means.

There is a branch of fuzzy logic, namely, *dispositional logic*, which is of particular relevance to the commonsense mode of reasoning which underlies our ability to communicate and make rational decisions in an environment of uncertainty and imprecision. Since classical predicate logic is not effective in such environments, a number of attempts have been made during the past decade to extend it in ways which would make it possible to address at least some of the problems which are associated with commonsense reasoning and, in particular, the problem of exceptions. The best known of the methods in this spirit are circumscription (3), nonmonotonic reasoning (4), and default reasoning (5).

The methods in question have played an important role in improving our understanding of commonsense reasoning and knowledge representation. However, based as they are on two-valued logic, they do not provide a framework for inference when the premises are fuzzy and/or are

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associated with fuzzy probabilities—which is characteristic of the premises representing commonsense knowledge. In this setting, dispositional logic provides an alternative system of inference based on fuzzy logic which is capable of addressing the issues of uncertainty and imprecision in the context of commonsense reasoning and knowledge representation. In what follows, we shall outline the conceptual structure of dispositional logic and provide a sketch of some of its main features. A more detailed exposition of a related theory of dispositions may be found in (7).

2. Dispositionality and Usuality

The point of departure in dispositional logic is the concept of a *disposition*, that is, a proposition which is preponderantly but not necessarily always true. For example, *birds can fly* is a disposition, as are the propositions *smoking is harmful*, *Swedes are blond*, *flying is safe*, *young men like young women*, and *it takes about an hour to drive from Berkeley to Stanford*. Most dispositions are of the form *A is B* or *A's are B's*, where A and B are fuzzy predicates, as in *smoking is harmful* and *Swedes are blond*. (Note that to be a Swede is a matter of degree.) Expressed in these forms, a disposition signifies that the conditional probability of B given A is high, where high should be interpreted as a fuzzy probability.

In more concrete terms, a disposition may be expressed in one of two canonical forms:

(a) *unconditional*:

$$\text{usually } (X \text{ is } A) \tag{1}$$

and (b), *conditional*:

$$\text{usually } (X \text{ is } A \text{ if } Y \text{ is } B), \tag{2}$$

where X and Y are variables, A and B are fuzzy predicates, and *usually* is a fuzzy quantifier which may be represented as a fuzzy number in the interval [0, 1]. In (1), A is a *usual value* of X, while in (2) B is a usual value of X conditioned on Y *is* B. Propositions of the form (1) and (2) are said to be *usuality-qualified*. In most cases, the quantifier *usually* in such propositions is implicit rather than explicit. As an illustration, the disposition *snow is white* may be represented in an unconditional form as

usually (Color (Snow) is white).

Similarly, the conditional proposition *snow is white if it is fresh* may be expressed as

usually (Color (Snow) is white if Snow is fresh),

where *Color (Snow)* plays the role of X; *white* the role of A; *Snow* the role of Y; and *fresh* the role of B. Note that in this case X is a function of Y.

The meaning of a disposition which is expressed in its unconditional form may be defined as follows (6). Assume for simplicity that X is a random variable which takes the values $u_1, \ldots, u_n$ with respective probabilities $p_1, \ldots, p_n$. Then, (1) may be interpreted as a constraint on the vector $p = (p_1, \ldots, p_n)$, with the degree, τ, to which p satisfies the constraint given by

$$\tau = \mu_{usually}\left(\textstyle\sum_i p_i \mu_A(u_i)\right), \quad i = 1, \ldots, n\,, \tag{3}$$

in which $\mu_{usually}$ is the membership function of the fuzzy number *usually*.

More generally, in the case of a conditional disposition, let p_{ij} and p_j denote, respectively, the joint probability that $X = u_i$ and $Y = v_j$, and the probability that $Y = v_j$, $i,j = 1, \ldots, n$. Then, the degree, τ, to which the constraint on p_{ij} and p_i is satisfied is given by

$$\tau = \mu_{usually} \left[\frac{\sum_{i,j} p_{ij} \left(\mu_A(u_i) \wedge \mu_B(v_j)\right)}{\sum_j p_j \mu_B(v_j)} \right] , \tag{4}$$

in which the argument of $\mu_{usually}$ is the conditional probability of the fuzzy event *X is A* given the fuzzy event *Y is B*.

Another important concept in dispositional logic is that of a ***subdisposition***, exemplified by ***slimness is attractive***. In this case, it would not be correct to interpret the proposition in question as ***most of those who are slim are attractive***. A more accurate interpretation would be that the conditional probability that a slim person is attractive is significantly higher than the unconditional probability that a person is attractive. More specifically, if a subdisposition is expressed as a conditional proposition of the form *X is A* if *Y is B*, then the relative increase, ρ, in the conditional probability of A given B may be expressed as

$$\rho = \frac{P(A \mid B) - P(A)}{1 - P(A)} . \tag{5}$$

To say that the conditional probability $P(A \mid B)$ is significantly higher than $P(A)$ is roughly equivalent to saying that the ratio ρ in (5) is a fuzzy number which is greater than or equal to ***medium***, which in turn is a fuzzy number close to 0.5. Under this assumption, the degree, τ, to which a given subdisposition constrains p_{ij} and p_j may be expressed as

$$\tau = \mu_\rho \left[\frac{p(A \mid B) - P(A)}{1 - P(A)} \right] , \tag{6}$$

in which $P(A \mid B)$ and $P(A)$ are given by

$$P(A \mid B) = \frac{\sum_{i,j} p_{ij} \left(\mu_A(u_i) \wedge \mu_B(v_j)\right)}{\sum_j p_j \mu_B(vu_j)} \tag{7}$$

and

$$P(A) = \sum_i p_i \mu_A(u_i) , \quad i = 1, \ldots, n ,$$

where p_i is the probability that $X = u_i$.

In dispositional logic, the concepts of a disposition and subdisposition as defined above provide a point of departure for the construction of a system of inference from commonsense knowledge. In this system, the techniques of fuzzy logic are employed to express commonsense knowledge in the form of usuality-qualified propositions exemplified by (1) and (2). Then, the deductive apparatus of both dispositional and fuzzy logic is called upon to provide answers to queries relating to the information stored in a database containing dispositions, subdispositions, and possibly other types of factual data.

The form of inference rules in dispositional logic may be illustrated by two basic rules: (a) the dispositional conjunctive rule; and (b) the dispositional *modus ponens*. These rules may be stated as follows.

The dispositional conjunctive rule

$$\frac{\begin{array}{c} usually\ (X\ is\ A) \\ usually\ (X\ is\ B) \end{array}}{(2\ usually \ominus 1)\ (X\ is\ A \cap B)}, \tag{8}$$

where $A \cap B$ denotes the intersection (or conjunction) of A and B, and the fuzzy quantifier $2\ usually \ominus 1$ is a fuzzy arithmetic expression in which $\ominus$ denotes the operation of subtraction in fuzzy arithmetic (2). As is generally true of deductions in dispositional logic, the fuzzy quantifier in the conclusion is less specific than the fuzzy quantifiers in premises. (What this means is that, viewed as a fuzzy subset of the unit interval, the fuzzy number *usuall* is a subset of $2\ usually \ominus 1$.)

The dispositional modus ponens

$$\frac{\begin{array}{c} usually\ (X\ is\ A) \\ usually\ (Y\ is\ B \text{ if } X\ is\ A) \end{array}}{usually^2\ (Y\ is\ B)}, \tag{9}$$

where $usually^2$ is the product of ***usually*** with itself in fuzzy arithmetic. As in the case of the dispositional conjunctive rule, the fuzzy quantifier $usually^2$ is less specific than ***usually***.

Simple examples of (8) and (9) are the following:

$$\frac{\begin{array}{c} usually\ (Age\ (Professor)\ is\ not\ very\ young) \\ usually\ (Age\ (Professor)\ is\ not\ very\ old) \end{array}}{(2\ usually \ominus 1)\ (Age\ (Professor)\ is\ not\ very\ young\ and\ not\ very\ old)}$$

$$\frac{\begin{array}{c} usually\ (Pressure\ is\ high) \\ usually\ (Volume\ is\ low\ if\ Pressure\ is\ high) \end{array}}{usually^2\ (Volume\ is\ low)}$$

Concluding remark. Although we have focused our attention in this note on the concept of dispositionality in the context of commonsense reasoning, its implications are much broader. Indeed, as we develop a better understanding of human reasoning we may discover that dispositionality—in its diverse manifestations—plays a key role in the remarkable human ability to make rational decisions in an environment of uncertainty and imprecision.

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